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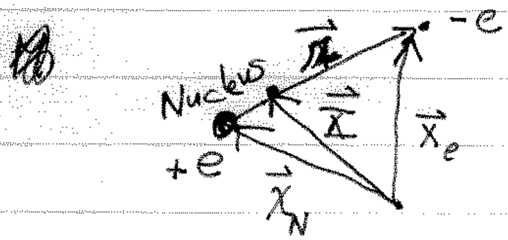
SQuInT Student Summer Retreat 6/25 - 6/29/01

Lecture #2: Optical Lattices

- Interaction of E+M field with atom:



Single electron in conduction shell (Alkali)

$$\vec{x} = \vec{x}_e - \vec{x}_N \qquad \vec{X} = \frac{m_e \vec{x}_e - m_N \vec{x}_N}{M}$$

$$\frac{\vec{p}}{\mu} = \frac{\vec{p}_e}{m_e} - \frac{\vec{p}_N}{m_N} \qquad \vec{P} = \vec{p}_e + \vec{p}_N$$

Interaction energy

$$H_{int} = \frac{e}{cm_e}\vec{p}_e \cdot \frac{\vec{A}}{c}(\vec{x}_e, t) - \frac{e}{cm_N}\vec{p}_N \cdot \frac{\vec{A}}{c}(\vec{x}_N, t)$$

$$-\vec{\mu}_e \cdot \vec{B}(\vec{x}_e, t) - \vec{\mu}_N \cdot \vec{B}(\vec{x}_N, t) + \cancel{\sum q_i V(\vec{x}_i, t)} \rightarrow 0$$

(Not static $\vec{E}$)

Plane wave (laser) $\vec{A}(\vec{x}, t) = \mathrm{Re}\left(\vec{\epsilon} A_0(\vec{x})\, e^{i(\vec{k}_L \cdot \vec{x} - \omega_L t)}\right)$

$$\vec{x}_e = \vec{X} + \frac{m_N}{m_e + m_N}\vec{r} \approx \vec{X} + \vec{r}$$

$$\vec{x}_N = \vec{X} - \frac{m_e}{m_e + m_N}\vec{r} \approx \vec{X}$$

$$\Rightarrow H_{int}^{(Field)} = \mathrm{Re}\left[\frac{e}{c}\left(\frac{\vec{p}_e}{m_e} e^{i\vec{k}_L \cdot \vec{r}} A_0(\vec{X} + \vec{r}) - \frac{\vec{p}_N}{m_N} A_0(\vec{X})\right) e^{i\vec{k}\cdot\vec{X}} e^{-i\omega_L t}\right]$$

Electric dipole approximation $\lambda_L \gg a_0 = |\vec{r}|$

$k_L a_0 \ll 1 \Rightarrow$ set $\vec{r} = 0$

$$\Rightarrow H_{int}^{(Field)} = \mathrm{Re}\left[\frac{e}{c}\left(\frac{p_e}{m_e} - \frac{P_N}{m_N}\right)\cdot\vec{\epsilon}A_0(\vec{X})\,e^{i(\vec{k}\cdot\vec{X}-\omega_L t)}\right]$$

$$= \frac{e}{c\,m_{rel}}\,\vec{p}\cdot\vec{A}(\vec{X},t)$$

field at center of mass!

Gauge transform.

$$= -e\vec{r}\cdot\vec{E}(\vec{X},t) \qquad \vec{E}_0 = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

$$= -\vec{d}\cdot\vec{E}(\vec{X},t)$$ electric dipole interaction

Note: In atom $d = e a_0$ $\quad \frac{\mu_e}{d} = \frac{v}{c} = \alpha = \frac{1}{137}$

$\Rightarrow$ Unless there is different external B (eg. magnetic trap), we can neglect magnetic interaction of $\vec{B}$-field in laser with atom $\quad \mu_N = \left(\frac{m_e}{m_N}\right)\mu_0$

$$\Rightarrow \boxed{H = \frac{\vec{p}^2}{2\mu} + \frac{\vec{P}^2}{2M} - \vec{d}\cdot\vec{E}(\vec{X},t)}$$

Note $\quad H_{int} \neq H_{rel} + H_{CM}$

$\Rightarrow$ CM and rel coord become entangled!

• Mechanical forces of light on atoms

Force acting on center of mass

$$\vec{F} = -\vec{\nabla} H_{int} = -\vec{\nabla}\left(\overline{\vec{d}(t)\cdot\vec{E}(\vec{X},t)}\right) \quad \text{time average}$$

(read as either classical E.O.M. or Heisenberg)

Consider ~~plane~~ monochromatic wave: $\vec{E}(\vec{X},t) = \mathrm{Re}\left(\underbrace{\vec{\epsilon}_L E_0(\vec{X})\, e^{i\phi(\vec{X})}}_{\tilde{\vec{E}}(\vec{X})} e^{-i\omega_L t}\right)$

$\vec{d}(t)$ determined by driving field

Lorentz oscillator $\vec{r}\uparrow$ e ω_0, Γ Steady state $\vec{r}(t) = \mathrm{Re}(\vec{\epsilon}\,\tilde{x}\, e^{-i\omega_L t})$

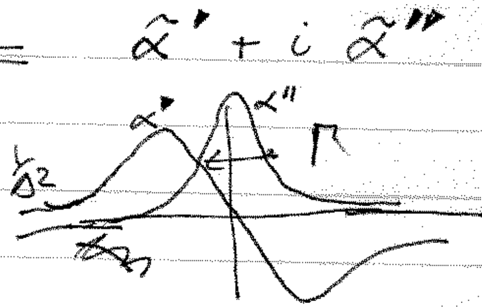
$$\ddot{\vec{r}} + \Gamma\dot{\vec{r}} + \omega_0^2\vec{r} = -\frac{e}{m_e}\vec{E}(\vec{x},t)$$

$$\tilde{x} = \frac{-e/m_e}{\omega_0^2-\omega_L^2-i\omega_L\Gamma}\, E_0(\vec{X})\, e^{i\Phi(\vec{X})}$$

$$\Rightarrow \tilde{d} = -e\tilde{x} = \tilde{\alpha}\, E_0(\vec{X})\, e^{i\Phi(\vec{X})}$$

Dynamic polarizability $\quad \tilde{\alpha} \cong \dfrac{\frac{e^2}{2m\omega_L}}{-\Delta - i\frac{\Gamma}{2}} = \tilde{\alpha}' + i\tilde{\alpha}''$

$\Delta \equiv \omega_L - \omega_0 \qquad \Delta \ll \omega_L$



$$\Rightarrow d(t) = \tilde{\alpha}' E_0(\vec{X})\cos(\omega_L t - \phi(\vec{X})) \quad \text{in-phase}$$

$$+\ \tilde{\alpha}'' E_0(\vec{X})\sin(\omega_L t - \phi(\vec{X})) \quad \text{in-quadrature}$$

$$\vec{F} = \overline{-\vec{\nabla}\left(d(t)\, E(\vec{x},t)\right)} = \overline{-d(t)\left[\vec{\nabla} E(\vec{x},t)\right]}$$

$$= -\frac{1}{2}\, Re\left(\tilde{d}^*\, \vec{\nabla}\tilde{E}(\vec{x})\right)$$

$$= -\frac{1}{2}\, Re\left[\tilde{d}^*\left(\vec{\nabla}E_0(\vec{x}) + i(\vec{\nabla}\phi)\, E_0(\vec{x})\right) e^{i\phi}\right]$$

$$= -\frac{1}{2}\alpha'\, E_0\left(\vec{\nabla}E_0\right) + \frac{1}{2}\alpha''\, E_0^2\left(\vec{\nabla}\phi\right)$$

$$\vec{F} = \vec{F}_{reactive} + \vec{F}_{dissipative}$$

$$\vec{F}_{reactive} = -\vec{\nabla}U$$

"dipole force" "gradient force"

$$\boxed{U = -\frac{1}{4}\alpha'\, E_0^2 \qquad \alpha' = \frac{e^2}{2m\omega_L}\left(\frac{-\Delta}{\Delta^2+\frac{\Gamma^2}{4}}\right)}$$

$\vec{F}_{dissipatn}$ = Radiation pressure

= Rate at which momentum is absorbed from field and transfered to atom

Recall $\frac{\vec{S}}{c^2} = \frac{\vec{E}\times\vec{B}}{4\pi c}$ = momentum flux density

Rate at which work is done by field on atom $\left\langle \frac{dW}{dt} \right\rangle = \tilde{\alpha}''\, E_0^2\, \omega_L$

Example: Plane waves $\phi = \vec{k} \cdot \vec{x}$

$$\vec{F}_{diss} = \frac{dW}{dt} \frac{\vec{k}_L}{\omega_L} = \frac{1}{c} \frac{dW}{dt} \hat{k}_L$$

Photons: $\frac{dW}{dt} = \hbar\omega_L \left(\frac{dN_{abs}}{dt}\right)$ $\quad p = \hbar k_L$

$$\Rightarrow \boxed{\vec{F}_{diss} = \left(\frac{dN_{abs}}{dt}\right) \hbar \vec{k}_L}$$ Scattering force

Note $\left(\frac{dN_{abs}}{dt}\right) = \left(\frac{dN^{spon}_{emit}}{dt}\right) = \frac{dN_{scat}}{dt} = \gamma_s \left(\begin{smallmatrix}\text{scattering}\\ \text{rate}\end{smallmatrix}\right)$ Cooling

"Light-shift potential"

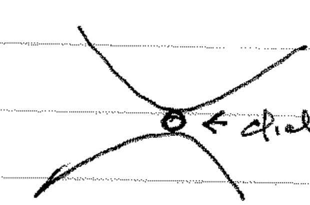
$$U = -\frac{1}{4} \alpha' |E_0(\vec{x})|^2 = -\frac{1}{2} \overline{\left(\vec{d}_{induced}(t) \cdot \vec{E}(\vec{x}, t)\right)}$$

$$\Delta \gg \Gamma \quad \alpha = \frac{-e^2}{2m\omega_L \Delta}$$

Red detuning $\Delta < 0$ $\quad \alpha > 0 \Rightarrow$ Strong field seeker

Blue detuning $\Delta > 0$ $\quad \alpha < 0 \Rightarrow$ Weak field seeker

"Optical tweezer"



$\Delta < 0$

Trapping atoms in a standing wave

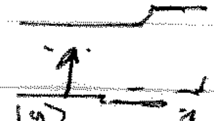
$U = -\frac{1}{2}\alpha' E_0^2(\vec{x})$ $\qquad \vec{F} \simeq -\vec{\nabla} I_0(\vec{x})$

$\uparrow$ intensity

Tight trap requires large gradient in $I_0 \Rightarrow$ standing wave!

Aside: Atom weakly excited $\equiv$ Lorentz oscillator (oscillator strength)

$U(\vec{x}) = \frac{1}{2}\hbar\Delta\, S(\vec{x})$



$\gamma_{scas} = N_e \Gamma = \frac{S(\vec{x})}{2}\Gamma$

$|g\rangle + \sqrt{\frac{S}{2}}|e\rangle \qquad S \ll 1$

$$S(\vec{x}) = \frac{\frac{I(\vec{x})}{I_s}}{1 + \frac{4\Delta^2}{\Gamma^2}}$$

Γ = natural line width $= \frac{1}{\tau_{life\text{-}time}}$

I_s = saturation intensity $= \hbar\omega_0\, \sigma_{abs}\, \Gamma$

($I = I_s \Rightarrow$ stimulate = spontaneous on resonance)

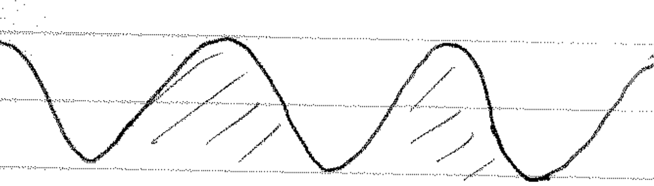
$\Delta \gg \Gamma$

Typical alkali $I_s \sim 1\ \frac{mW}{cm^2}$

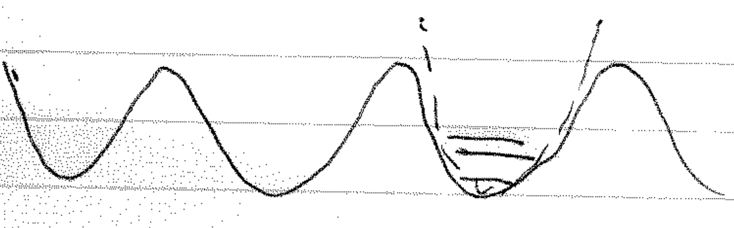
$$U(\vec{x}) = \frac{\hbar\Gamma^2}{8\Delta}\frac{I(\vec{x})}{I_0} = \underbrace{\boxed{\frac{\hbar\Gamma^2}{8\Delta}\frac{I_{max}}{I_0}}}_{U_{pp}} \sin^2 k_L z$$

(1D standing wave)

$I(x)$



$U(x)$



$U(\vec{x}) \approx \frac{1}{2} M\omega_{osc}^2 z^2$

$= \quad (k_L z)^2$

$$\Rightarrow \omega_{osc} = 2\frac{\sqrt{U_p E_R}}{\hbar} \qquad E_R = \frac{(\hbar k)^2}{2M} = \text{recoil energy}$$

$= 200\,nK$ for ~~Cesium~~ ^{133}Cs

$$U_{pp} = \frac{\hbar\Gamma}{8}\left(\frac{\Gamma}{\Delta}\right)\frac{I_{max}}{I_s} \qquad \hbar\gamma_s^{max} \sim \frac{\hbar\Gamma}{4}\left(\frac{\Gamma}{\Delta}\right)^2\frac{I_{max}}{I_s}$$

$$\frac{U_{pp}}{\hbar\gamma_s} = \frac{\Delta}{\Gamma}$$

#'s: $I_{max} = 10^6 I_s$ (1 W, 10 μm spot)

$\Delta = 10^4\Gamma$

$$S_{max} = \frac{1}{4}\left(\frac{\Gamma}{\Delta}\right)^2\frac{I_{max}}{I_s} = \frac{1}{4}\times 10^{-2}$$

$$U_{pp} = \frac{1}{2}\hbar\Delta S_{max} = \frac{\hbar\Gamma}{8}\left(\frac{\Delta}{\Gamma}\right)\times 10^{-2} = \frac{100}{8}\hbar\Gamma$$

$$\approx 10\,\hbar\Gamma$$

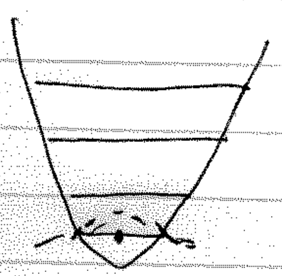
$$\frac{\Gamma}{2\pi} = 5\,MHz \Rightarrow \hbar\Gamma \cong 5\,\text{mKelvin}$$

Oscillation frequency: $\omega_{osc} = 2\sqrt{10\,\frac{1}{2600}} = \frac{2}{\sqrt{260}} = \frac{1}{8}\Gamma$

≈ 500 kHz

Scattering rate $\gamma_s \sim \frac{1}{8}10^{-2}\,\Gamma \sim$

$$\frac{\omega_{osc}}{\gamma_s} \sim 10^2$$



Cooling to the ground state

blue detuning

$\gamma_s \to \eta^2\gamma_s$

$\downarrow$

3×10^{-3}

$$k\Delta z_0 = \sqrt{\frac{\hbar k^2}{2m\omega_{osc}}} = \sqrt{\frac{E_R}{\hbar\omega_{osc}}} = \eta$$

Our ~~problem~~ example: $\eta = \sqrt{\frac{8E_R}{\hbar\Gamma}} = \sqrt{8/2600} = 0.05$

Lamb-Dicke effect: (Mossbauer)

Photon scattering matrix element

$$\left| \langle \psi_f | H_{AV}^{(-)} H_{AL}^{(+)} | \psi_i \rangle \right|^2 \gamma_s$$

$$|\psi_i\rangle = |g\rangle \otimes |\Phi_{cm}^{(i)}\rangle \qquad |\psi_f\rangle = |g\rangle \otimes |\Phi_{cm}^{(f)}\rangle$$

red-detuned: $H_{AL}^{(+)} = |e\rangle\langle g| \, e^{ikz}$ $\qquad H_{AV}^{(-)} = |g\rangle\langle e| \, e^{-ikz}$

$$\Rightarrow \langle \Phi_{cm}^{(f)} | e^{-ik\hat{z}} | \Phi_{cm}^{(i)} \rangle \gamma_s$$

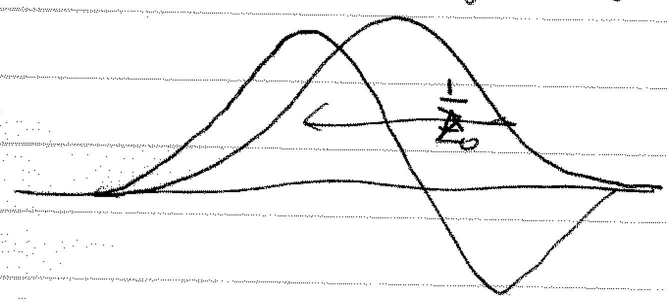
momentum translation $\qquad \eta << 1$

$$\left| \langle \Phi_{cm}^{(f)} | 1 - i\eta\left(\frac{\hat{z}}{z_0}\right) + \dots | \Phi_{cm}^{(i)} \rangle \right|^2$$
$$\left| \langle n_f | \qquad\qquad | n_i \rangle \right|^2 \gamma_s$$

$$= \left(\delta_{n_i, n_f} + \eta^2 \delta_{n_f, n_i \pm 1} + (\,)\eta^4 \delta_n \right.$$

Heating / inellastic scattering to same $|g\rangle$

suppress by η^2

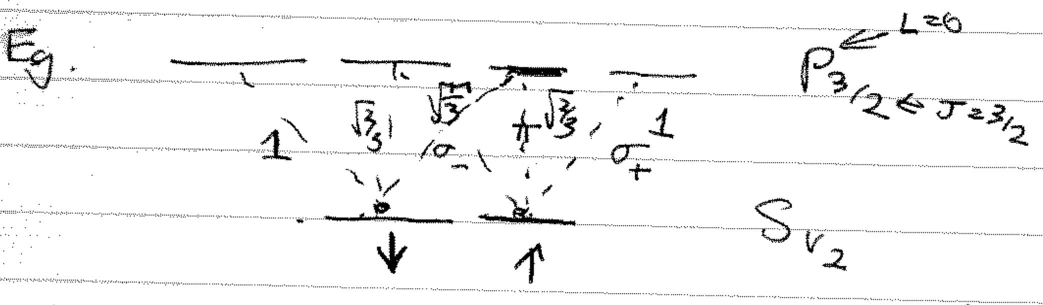


p space

Tensor polarizability

Atom responds differently to different polarizations

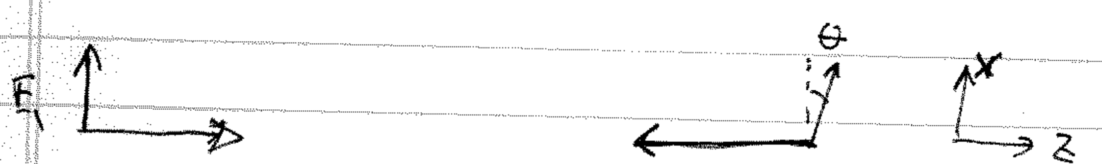
Eg.



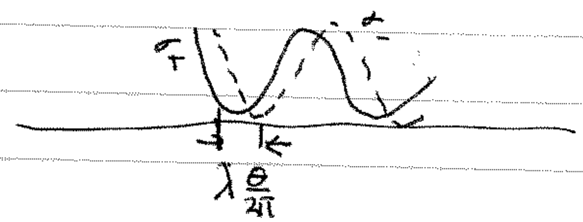
Light shift: $\hat{U} = -\frac{1}{4}\vec{E}^{(+)}_{(\vec{x})} \cdot \hat{\overleftrightarrow{\alpha}} \cdot \vec{E}^{(-)}_{(\vec{x})}$

$$= \sum_m U_m |m\rangle\langle m| + \sum_{m \neq m'} U_{mm'} |m\rangle\langle m'|$$

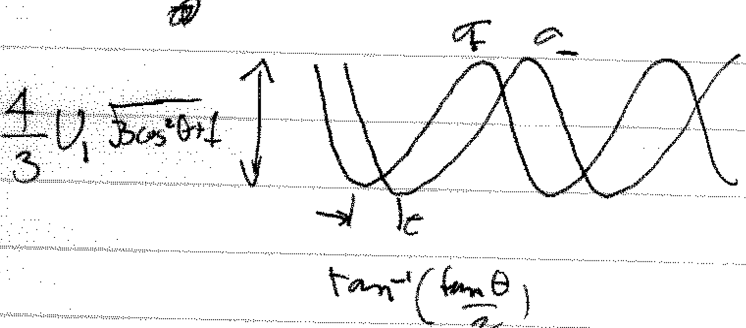
Lin $\measuredangle$ Lin



$$\vec{E} = \sqrt{2}\, E_1 \left\{ -e^{-i\theta/2} \cos(k_L z + \theta/2)\, \vec{e}_+ + e^{i\theta/2} \cos(k_L z - \tfrac{\theta}{2}) \vec{e}_- \right\}$$



$U_\uparrow = U_+ + \frac{1}{3} U_-$ $\qquad$ $U_\downarrow = U_- + \frac{1}{3} U_+$



No off diagonal

Effective B-field

$$\hat{U} = U_0 \hat{\mathbb{1}} + \vec{B}_{fict} \cdot \vec{\sigma} \qquad 2\times 2$$

$$U_0 \sim \vec{E}^* \cdot \vec{E}$$

$$B_{fic} = \frac{\vec{E}^* \times \vec{E}}{i}$$

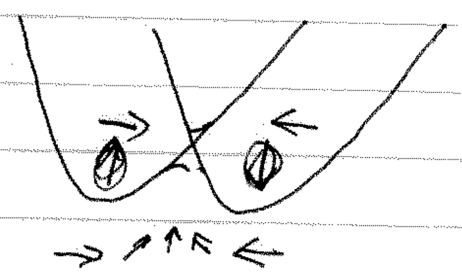
$$U_0 \sim \cancel{\cos k} \cos\theta \cos(2kz)$$

$$B_{eff} \sim \sin\theta \sin(2kz)\, \vec{e}_z$$

$$H = \frac{p^2}{2m} + U_0 + \vec{\mu} \cdot \vec{B}_{eff}$$

$$\vec{B}_{eff} = \vec{B}_{fict} + \vec{B}$$

E.g.



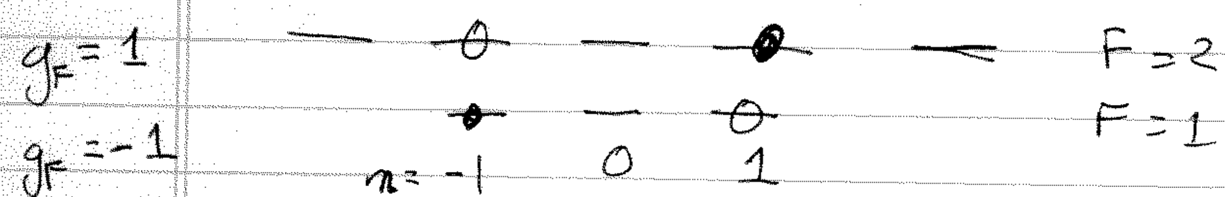
Entangled spin wave packets

With Hyperfine $\Delta \gg \Delta E_{HF}$

$$\hat{U} = U_0 \hat{\mathbb{1}} + g_F \vec{\mu}_B \cdot \vec{B}$$

$\uparrow$ Landé-g

Qubit encoding:



e.g. ^{87}Rb

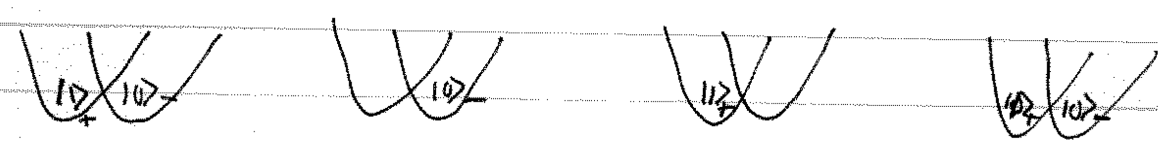
DFS

$|0\rangle_+ =$ $\qquad$ $|0\rangle_- =$

$|1\rangle_+ =$ $\qquad$ $|1\rangle_- =$

Dipole-dipole: Scalling argument
Light shift

Read-out: Ensemble



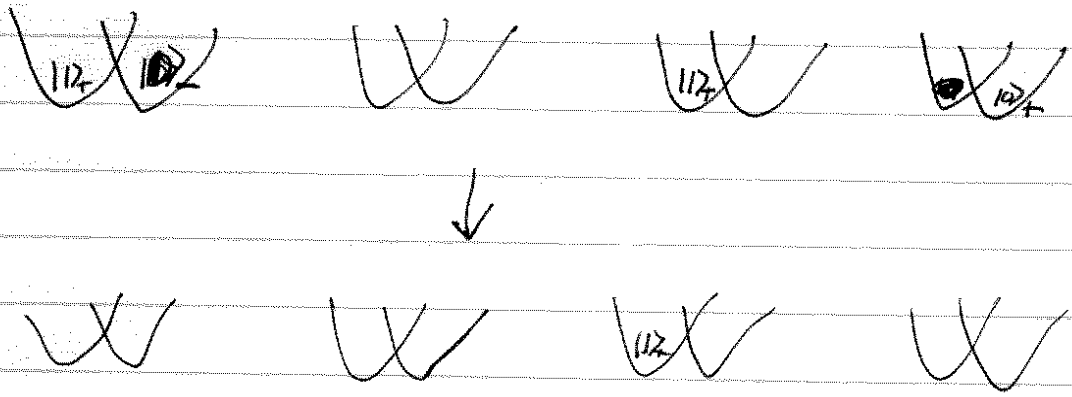
Types of errors: (i) the control ~~or~~ and/or target end up in ~~wrong out comp basis~~ wrong st.

(ii) ~~Both~~ one or the other go out of ~~cmp~~ basis

(i) end in $|0\rangle_I \Rightarrow F_\downarrow$

(ii) Flush everything in $|0\rangle$, repeat

- error Probability P
- Start with N pairs $\Rightarrow$ $(1-P)N$ pairs, $\alpha P N$ new upaired target



Full process tomography

Start with N pairs

After 1st gate $N(1-P)$ pairs $+$ $\alpha N P$ unpaired targets

After 2nd gate $N(1-P)^2$ pairs $\alpha P(1-P)N$ unpaired targets

See how # of targets decrease

$$\hat{\rho} \rightarrow \sum_{\alpha} A_{\alpha}^{\dagger} \hat{\rho} A_{\alpha}$$