What is Quantum Information and Technology?

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Outline

- What is **classical information**?
- What is **quantum information**?
- What is **quantum computing**?
- How would we **implement** these ideas?

References:

- M. A. Nielsen and I. Chuang, *Quantum computation and quantum information"* (Cambridge Press, 2000).
- Prof. Preskill's notes: http://www.theory.caltech.edu/people/preskill/ph229/
- *Introduction to Quantum Computation and Information*, H-K Lo, S. Popescu, T. Splliller eds., (World Scientific, 1998).
- Special Issue of Physical Implementations: Fortschritte der Physik **48** 2000.


WHEEL OF FORTUNE


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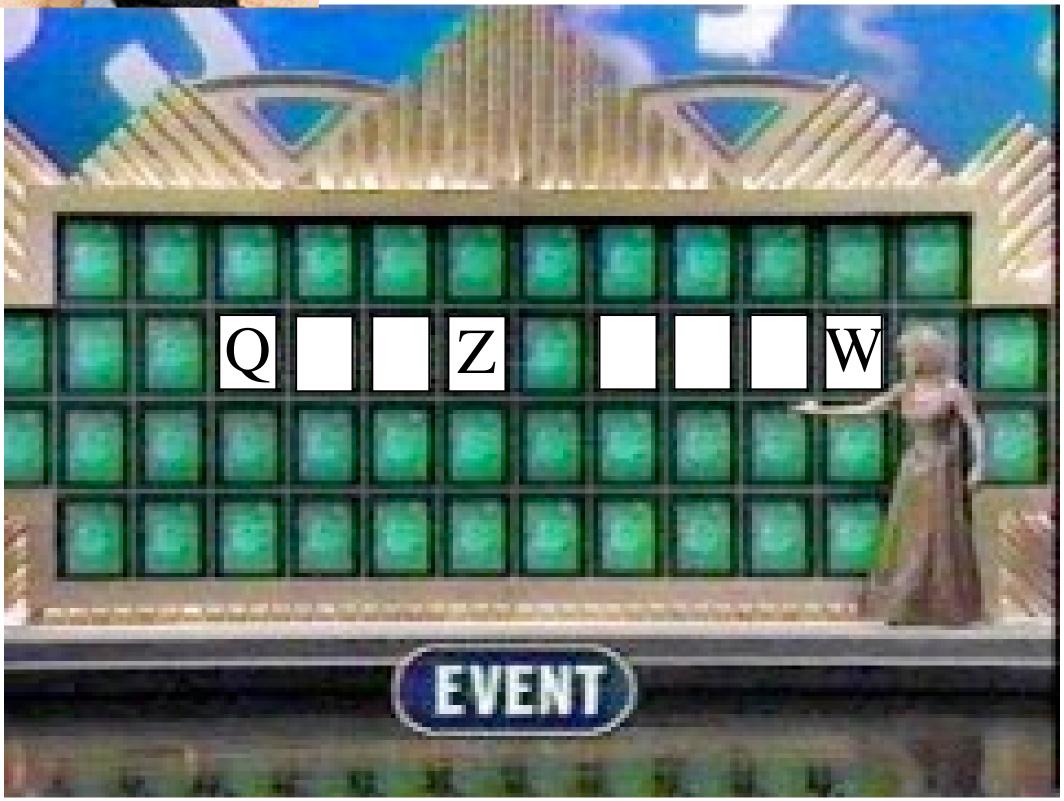
WHEEL OF FORTUNE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Quantifying Information

3 bit message: {000, 001, 010, 011, 100, 101, 110, 111}

With no *prior knowledge*:

$$Information = \log_2(\#\text{ of possibities}) = \log_2(8) = 3 \text{ bits}$$

With *a priori knowledge (probabilistic)*:

Probability 0 = p
Probability 1 = 1-p

- "Typical word" of length N has: Np 0's and N(1-p) 1's
- $$\#_{typ} = \binom{N}{Np} = \frac{N!}{(Np)!\left[N(1-p)\right]!}$$
- Information in a typical word = $\log_2(\#_{typ})$

Shanon Information

Shanon Information = ***Average information/ letter***

$$H(p) = \frac{\log_2(\#_{typ})}{N} = -p\log_2(p) - (1-p)\log_2(1-p)$$

(Sterling's approximation: $\log(N!) \approx N\log N - N$)

Generally, given alphabet: $\{a_j \mid j = 1,..,n\}$ with probability p_j

$$H(A) = -\sum_{j=1}^{n} p_j \log_2(p_j) = \textbf{\textit{Entropy}}$$

$-\log_2(p_j) =$ Information in a_j

$0 \le H(A) \le \log_2(n)$

Mutual Information

Suppose: Alice send message ***A*** with probability ***p(A)***

Bob receives message ***B*** with probability ***p(B)***

How much information can Alice communicate to Bob?

Degree of correlation: $p(A;B) = p(A)p(B) \div p(A,B)$

Joint probability

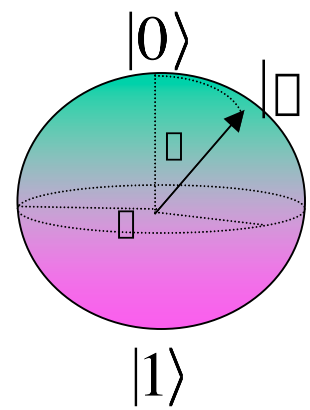
$$I(A;B) = \langle -\log(p(A;B)) \rangle = I(B;A)$$

- **Information Bob gets from Alice**
- **Information common to *A* and *B***

Quantum Information

Information encoded in a *quantum state*, $|\psi\rangle$, more generally ρ.

Qubit: Two-level quantum system



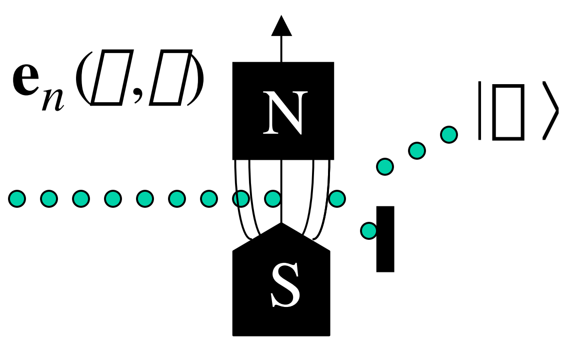
$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle$

$\{|0\rangle, |1\rangle\}$
"Logical Basis"

Bloch sphere

Tremendous amount of information ***encoded*** in $|\psi\rangle$

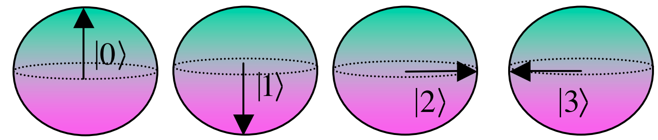
$\mathbf{e}_n(\theta,\varphi)$



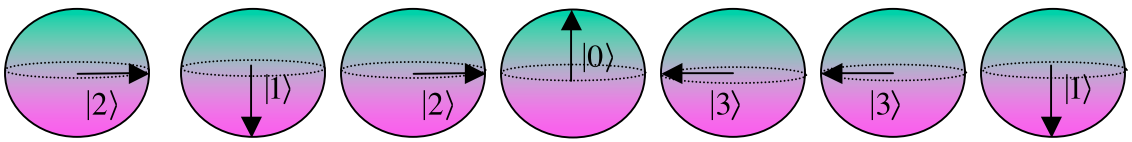
Given finite precision to specify θ,φ Alice can specify *many* different $|\psi\rangle$

Inaccessible Information

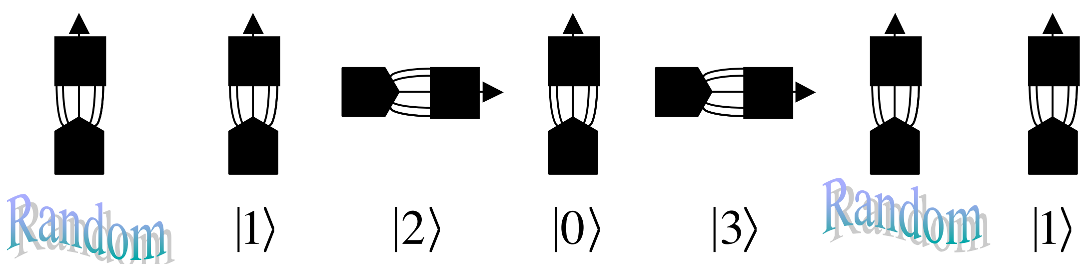
- Alice encodes 2 " classical bit" in a qubit



<u>Message</u>



- Bob decodes through Stern-Gerlach apparatus.

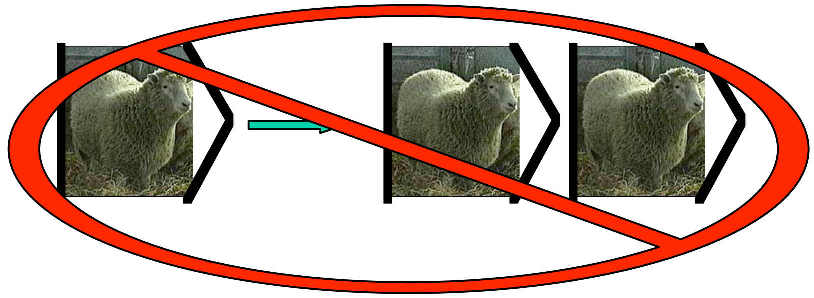


Quantum information ***cannot be read***

Holevo Bound
Alice cannot send more than one bit of information to Bob per qubit!

$$I(A;B) \leq 1 \text{ bit}$$

No Cloning



Suppose Bob wants to be cleaver and make many copies of $|\psi\rangle$.

$$|\psi\rangle \rightarrow |\psi\rangle|\psi\rangle|\psi\rangle|\psi\rangle|\psi\rangle|\psi\rangle$$

e.g. copy the unknown qubit onto an "ancilla" qubit $|\psi\rangle|0\rangle \rightarrow |\psi\rangle|\psi\rangle$

- Transformation on basis states

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle \qquad |1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$

- ***Linearity***

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle = \alpha|0\rangle|0\rangle + \beta|1\rangle|0\rangle \rightarrow$$

$$\alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle \neq (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)$$

Quantum Information cannot be copied!

Information Gain ⇒ *Disturbance*

Attempt to copy “distinguishing information” into an ancilla.

$$|\psi\rangle|u\rangle \to |\psi\rangle|v\rangle \qquad |\varphi\rangle|u\rangle \to |\varphi\rangle|v'\rangle$$

$$(\langle\varphi|\langle u|)(|\psi\rangle|u\rangle) = (\langle\varphi|\langle v'|)(|\psi\rangle|v\rangle)$$

$$\langle\varphi|\psi\rangle = \langle\varphi|\psi\rangle\langle v'|v\rangle$$

$$\text{If } \langle\varphi|\psi\rangle \neq 0 \;\Rightarrow\; 1 = \langle v'|v\rangle$$

Any attempt to distinguish between two ***non-orthogonal*** states necessarily results in a disturbance of the states

Composite Systems

Classical

- One bit: $A=\{a \mid a=0 \text{ or } 1\}$
- Two bits: $A \otimes B = \{(a,b) \mid a,b = 0 \text{ or} 1\}$ ***Cartesian product***
- N bits: Space has 2^N configurations.

Quantum

- One qubit:

$$A = \{|\psi\rangle,\ |\psi\rangle = c_0|0\rangle + c_1|1\rangle,\ \ c_0, c_1 \text{ finite precision complex numbers}\}$$

- Two qubits: ***Tensor product***

$$A \otimes B = \{|\psi\rangle_{AB} = c_{00}|0,0\rangle + c_{01}|0,1\rangle + c_{10}|1,0\rangle + c_{11}|1,1\rangle\}$$

- N qubits: Space has $(2^x)^{2^N}$ states, with x bits for each c.

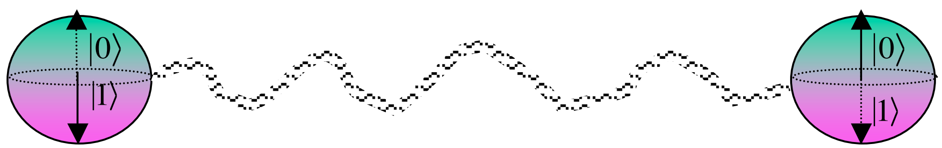
Expontenial growth in state space

Entanglement

(Pure) states *N*-qubits are generally "*entangled*"

$$|\psi\rangle \neq |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_N$$

→ ***Nonclassical correlations***
e.g. Two qubits, Bell's Inequalities



$$|\Psi^{(-)}\rangle = \left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B\right)/\sqrt{2}$$

Alice receives a random bit

Bob receives a random bit

Alice and Bob cannot communicate even one bit

- **Quantum-Information cannot be read.**
- **Quantum-Information cannot be copied.**
- **Nonorthogonal states cannot be distinguished.**
- **Exponential growth in *inaccessible* information.**
- **Quantum correlations cannot be used for communicating classical information**
- **Measurement is *irreversible - collapse of the wave function*.**

Quantum Mechanics is a Nuisance?

Quantum Information as a Resource

Information-Gain/Disturbance:

- Quantum Cryptography - Secret key distribution

Tensor Structure of Composites (Entanglement)

- Quantum dense coding - Sending two classical bits with one classical bit plus EPR.
- Quantum teleportation - Communicating a qubit
- Quantum Computation

Elements of Quantum Computation

$$|\psi_{in}\rangle \longrightarrow \boxed{\hat{U}} \longrightarrow |\psi_{out}\rangle$$

Take advantage of exponentially large state space

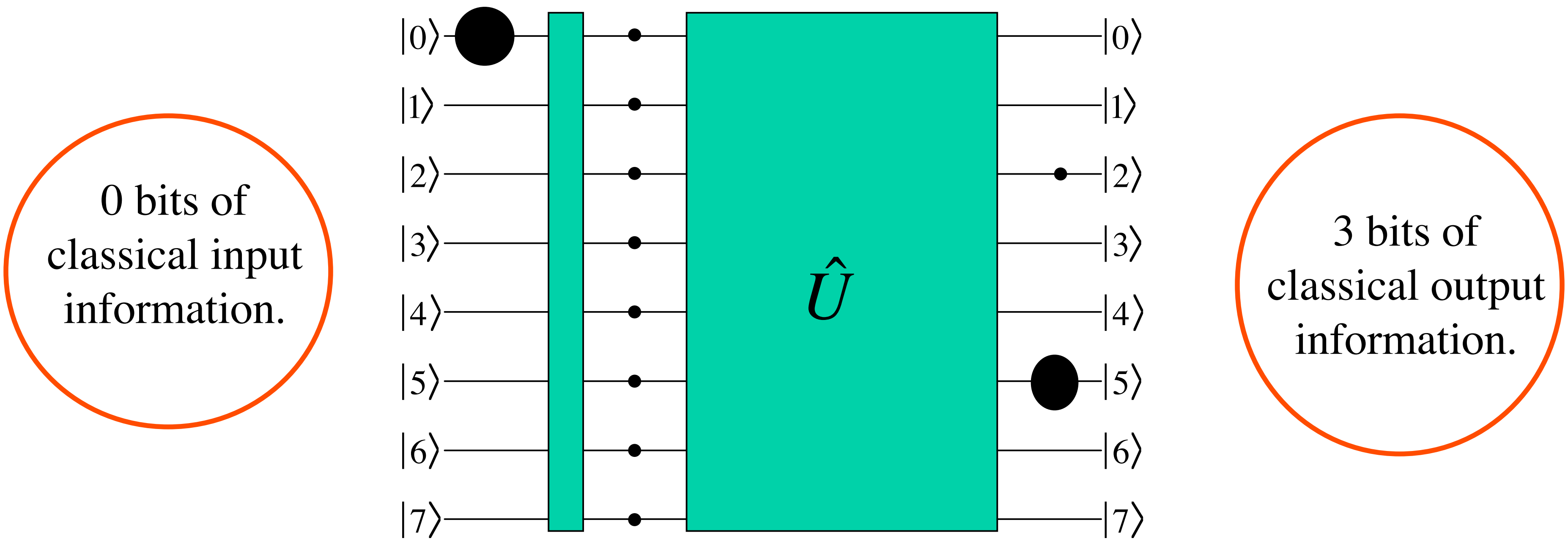
- ***Quantum Register:*** quantum state of the system
- ***Quantum Logic Gates:*** unitary transformation on subsystems
- ***Error-tempering:*** correct/suppress errors
- ***Measurement:*** read out classical information

Quantum Parallelism (Deutsch)

E.g. 3-qubit "quantum register"

$$|0\rangle = |0\rangle|0\rangle|0\rangle \quad |1\rangle = |0\rangle|0\rangle|1\rangle \quad |2\rangle = |0\rangle|1\rangle|0\rangle \quad |3\rangle = |0\rangle|1\rangle|1\rangle$$

$$|4\rangle = |1\rangle|0\rangle|0\rangle \quad |5\rangle = |1\rangle|0\rangle|1\rangle \quad |6\rangle = |1\rangle|1\rangle|0\rangle \quad |7\rangle = |1\rangle|1\rangle|1\rangle$$



Quantum Computer is a multiparticle Interferometer

Quantum Computing: Quantum Control in Asymptopia

$$|\square_{in}\rangle \longrightarrow \boxed{\hat{U}} \longrightarrow |\square_{out}\rangle$$

$$|\square\rangle \equiv \textit{Many-body State}$$

Complexity: *Asymptotic Behavior*

Given n bits to specify the state, how do the resources scale as n | |

Resources:

- Time
- Energy
- Space

- "***Easy***" Polynomial in *n*
- "***Hard***" Exponential in *n*

Quantum Logic Gates

- **Single qubit:**

$$U = \sum_{i=0}^{3} c_i \sigma_i, \quad \sum |c_i|^2 = 1$$

$$\sigma_0 = 1, \quad \sigma_{i=1,2,3} = \text{Pauli}$$

Rotation on Bloch sphere

NOT: $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad |0\rangle \to |1\rangle$, $\quad |1\rangle \to |0\rangle$

Hadamard: $H = \dfrac{\sigma_1 + \sigma_3}{\sqrt{2}}$ $\quad |0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2}$, $\quad |1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2}$

- **Two qubit:**

$$U = \sum_{i,j} c_{ij}\, \sigma_i^{(1)} \sigma_j^{(2)}$$

"Entangling unitary"

Controlled NOT: $|x\rangle|y\rangle \to |x\rangle|x \oplus y\rangle$

Control Target

$|0\rangle|0\rangle \to |0\rangle|0\rangle$

$|0\rangle|1\rangle \to |0\rangle|1\rangle$

$|1\rangle|0\rangle \to |1\rangle|1\rangle$

$|1\rangle|1\rangle \to |1\rangle|0\rangle$

Flip the state of the target bit conditonal on the state of the control bit $\quad \sigma_3 \quad \sigma_1$

$$\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle \to \frac{|0\rangle|0\rangle + |1\rangle|1\rangle}{\sqrt{2}}$$

Universality

• <u>Logic Gates:</u> Basic building blocks

$\hat{U}: \mathrm{H} \equiv \Box\, a_{i_1 i_2 \cdots i_N} \Box_{i_1} \quad \Box_{i_2} \quad \cdots \quad \Box_{i_N}$

Unitary acts on combinations of qubits

Universal Logic Gates

$\left\{ R_{\mathbf{e}_n}(\Box);\ \ cNOT \right\}$

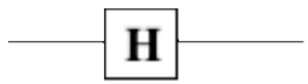
Single qubit rotation

Two-qubit entangling gate

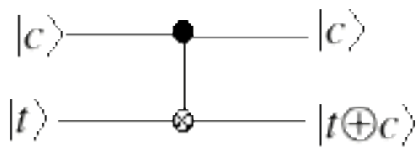
Entangling Gate: $\hat{H}_{12} \neq \hat{H}_1 + \hat{H}_2$ Nonseparable

Efficient algorithms: # of gates not exponential in N

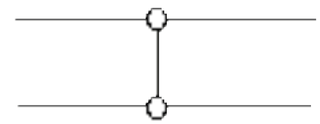
Quantum Circuits



Hadamard

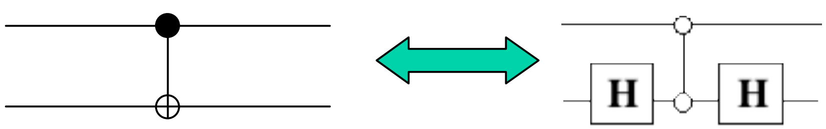


CNOT

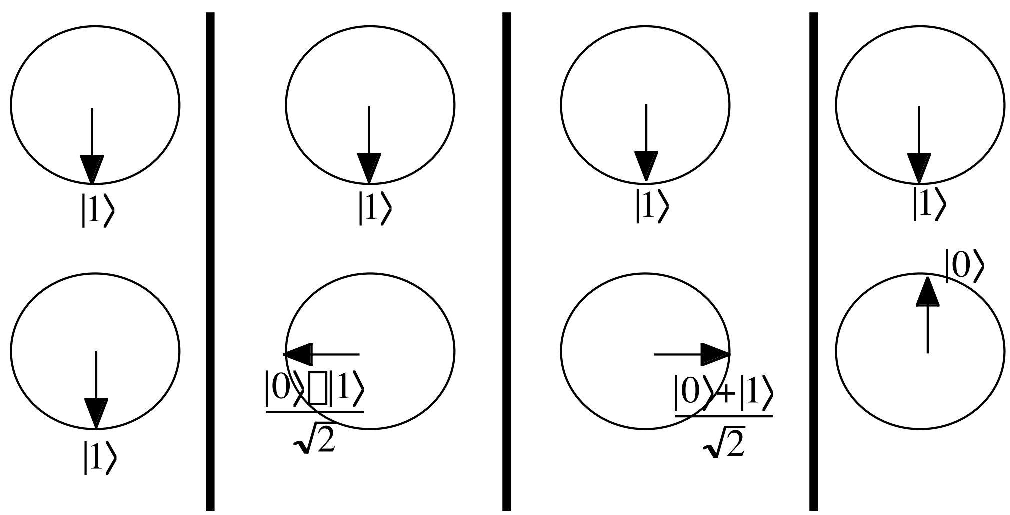


CPHASE

$$|0\rangle|0\rangle \to |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \to |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \to |1\rangle|0\rangle$$
$$|1\rangle|1\rangle \to -|1\rangle|1\rangle$$



CNOT from CPHASE



(I) **(II)** **(III)** **(IV)**

Deutsch's Problem

- **Problem:** Given function with two inputs and outputs

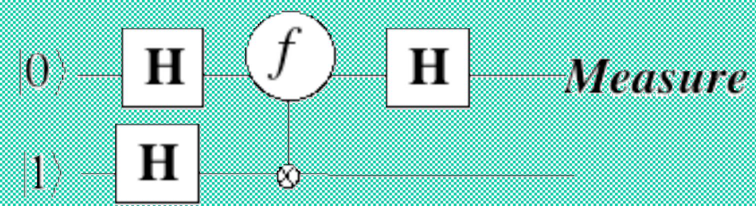
$$f:\{0,1\}\rightarrow\{0,1\} \quad \text{is} \quad f(0)=f(1)\ ?$$

- **Classical solution - requires TWO calls of *f***
- **Quantum solution - requires ONE call of *f***

Quantum function evaluation

$$\hat{U}_f|x\rangle|0\rangle = |x\rangle|f(x)\rangle$$

REVERSABLE



$$|0\rangle|1\rangle \Rightarrow \left(|0\rangle+|1\rangle\right)\left(|0\rangle-|1\rangle\right) = \left(|0\rangle\left(|0\rangle-|1\rangle\right)+|1\rangle\left(|0\rangle-|1\rangle\right)\right)$$

$$\Rightarrow (-1)^{f(0)}|0\rangle\left(|0\rangle-|1\rangle\right)+(-1)^{f(1)}|1\rangle\left(|0\rangle-|1\rangle\right)$$

$$=\left(|0\rangle+(-1)^{f(1)-f(0)}|1\rangle\right)\left(|0\rangle-|1\rangle\right)$$

$$\Rightarrow |0\rangle\left(|0\rangle-|1\rangle\right) \qquad \textbf{\textit{OR}} \qquad \Rightarrow |1\rangle\left(|0\rangle-|1\rangle\right)$$

(f constant) ***(f balanced)***

The Tao of Quantum Computation

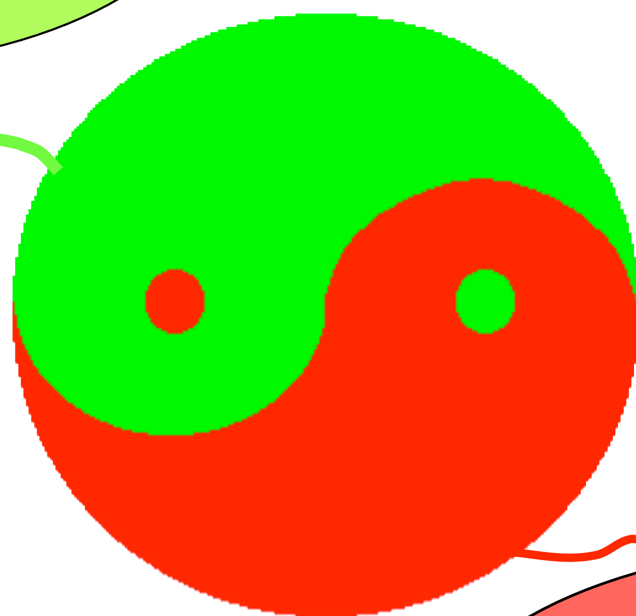
Coupling Between Qubits

- Entanglement

Coupling To External Drive

- Unitary evolution

Coherence



Decoherence

Coupling to the Environment

Errors

Physical Implementations

Atomic-Molecular Optical Systems (Gas Phase)

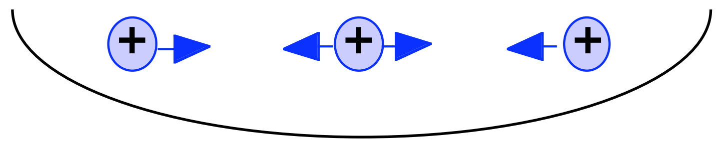
- Ion Traps
- Cavity *QED*
- Neutral Atom Traps
- Linear/Nonlinear Optics

Condensed Matter (Liquid or Solid Phase)

- Semiconductors (electronics)
- Nuclear/Electron Magnetic Resonance (liquid, solid, spintronics)
- Superconductors (flux or charge qubits)
- Electrons floating on liquid helium

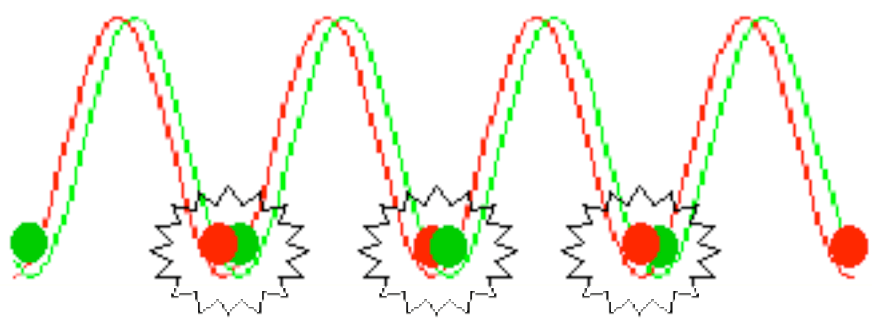
Electric Monopole

- **Ion Trap**



- **Strong Qubit Coupling: Coulomb Repulsion**
- **Strong Coupling to Environment - Technical Noise**

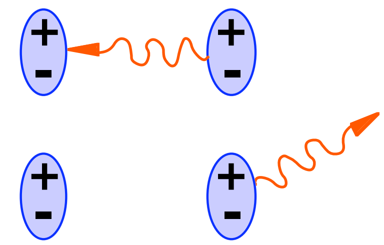
- **Neutral atom trap (optical lattice)**



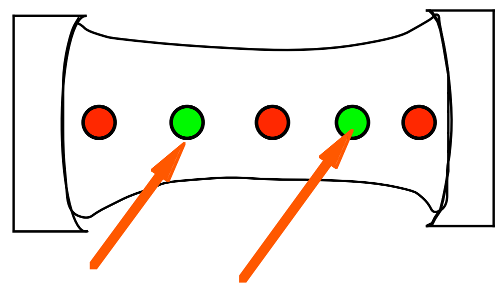
- **Strong Qubit Coupling "On demand:** ***elastic collision***
- **Coupling to Environment -** ***inelastic collision***

Electric Dipole-Dipole

- Coherent photon exchange
- Spontaneous emission

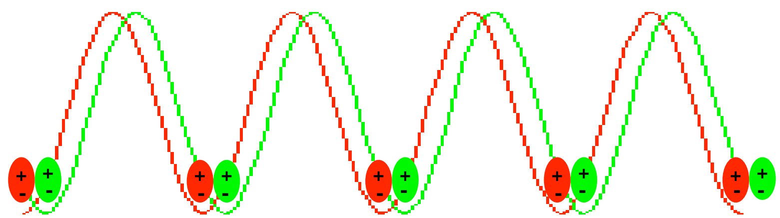


Cavity QED:



- Interactions turned “on” and “off”
- Real photon exchange
- Strong-coupling regime (enhance coherence)

Optical Lattice:



- Virtual photon exchange
- Near-field interaction dominates

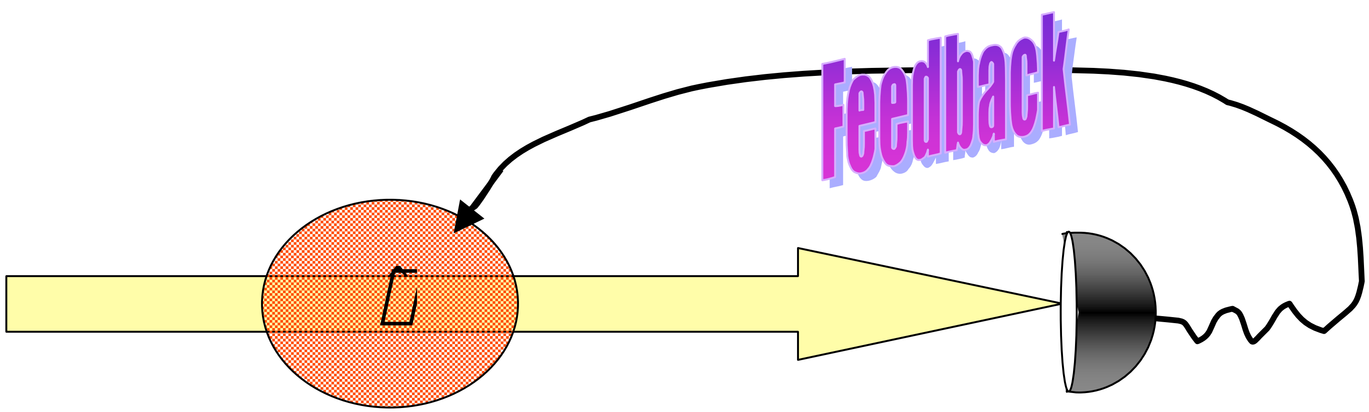
Measurement

Ideal: $|\psi_{in}\rangle$ → $\hat{U}$ → $|\psi_{out}\rangle$

Reality: $\hat{\rho}_{in}$ → **$** → $\hat{\rho}_{out}$

$$\$[\hat{\rho}] = \sum_j \hat{A}_j \hat{\rho} \hat{A}_j^\dagger$$

$$\sum_j \hat{A}_j^\dagger \hat{A}_j = 1$$



Summary

- Information Processing constrained by ***physical laws***.
- Quantum Information:
  - **Information-gain/disturbance.**
  - **Exponential growth of state space. Entanglement.**
- Quantum Computation - asymptotic savings of ***physical resources***.
- Physical Implementation - *Quantum Control of Many-body System!*

Classical Physics

Digital: Particles

Analog: Waves

→

Quantum Physics

Waveparticle

Duality