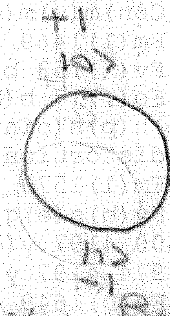
SQuInT Network Retreat, St. John's College, 01-6-25

Entanglement



Bipartite pure states

Example: Bell states

A B

$$|\psi^{AB}\rangle \neq |\psi^{A}\rangle \otimes |\psi^{B}\rangle$$

$$|\Phi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\Psi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

⓪ Schrödinger: Max info about whole is not max info about parts.

Marginal entropy: $\rho_A = \mathrm{tr}_B(|\psi^{AB}\rangle\langle\psi^{AB}|)$

$$S(\rho_A) = S(\rho_B)$$

Who cares? $0 \le S(\rho_A) \le \log d$

① Bell inequalities: Nonlocal correlations

GHZ $\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$

$\sigma_x \otimes \sigma_y \otimes \sigma_y$ +1

$\sigma_y \otimes \sigma_x \otimes \sigma_y$ +1

$\sigma_y \otimes \sigma_y \otimes \sigma_x$ +1

$$+1 = \underbrace{m_x^{(1)} m_y^{(2)} m_y^{(3)}}_{+1}\,\underbrace{m_y^{(1)} m_x^{(2)} m_y^{(3)}}_{+1}\,\underbrace{m_y^{(1)} m_y^{(2)} m_x^{(3)}}_{+1} \overset{?}{=} m_x^{(1)} m_x^{(2)} m_x^{(3)}$$

↓

$$\sigma_x \sigma_y^2 \otimes \sigma_y \sigma_x \sigma_y \otimes \sigma_y^2 \sigma_x = -\sigma_x \otimes \sigma_x \otimes \sigma_x$$

$\sigma_x \otimes \sigma_x \otimes \sigma_x$ −1

Teleportation

Shared entanglement
2 bits
No cloning

Q0: What is teleported?

Multipartite pure states:

Bipartite mixed states: ρ^{AB}

Product states: $\rho^{AB} = \rho^{A} \otimes \rho^{B} = \sum_{ij} \mu_i \nu_j |e_i\rangle\langle e_i| \otimes |f_j\rangle\langle f_j|$

Eigendecomposition: $\rho^{AB} = \sum_i \lambda_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}|$

Werner state (qubits): $\rho^{AB} = F|\Phi^{(-)}\rangle\langle\Phi^{(-)}| + \frac{1-F}{3}\left(1 - |\Phi^{(-)}\rangle\langle\Phi^{(-)}|\right)$

$$\rho^{A} = \tfrac{1}{2}1 = \rho^{B}$$

Separability: $\rho^{AB} = \sum_i p_i |\psi_i\rangle\langle\psi_i| \otimes |\phi_i\rangle\langle\phi_i|$

$$F \leq \tfrac{1}{2}$$

Q1: Find an ensemble for $F \leq \frac{1}{2}$

Use rotational invariance
look at correlation
matrix

Entanglement of formation: $\frac{m}{n} \log d$

in max ent

out ρ^{AB}

$$E(\rho^{AB}) = \min\left\{ \sum_i p_i S(\rho_i^{A}) \,\middle|\, \rho^{AB} = \sum_i p_i |\psi_i^{AB}\rangle\langle\psi_i^{AB}| \right\}$$

out max ent

Entanglement of distillation: $\frac{n}{m} \log d$

in ρ^{AB}

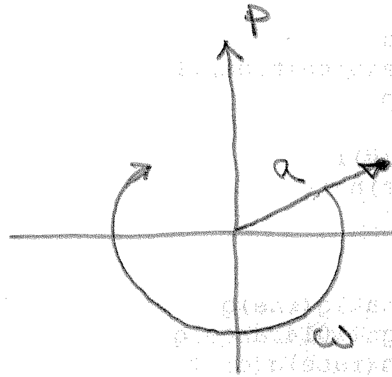
Major ?: nonlocal correlations vs. entanglement

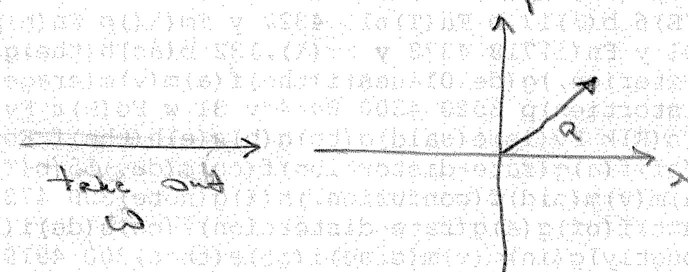
Continuous systems: HO, x and p

$$a = \frac{1}{\sqrt{2}}(x + ip)$$

Classical

$$a^{\dagger} = \frac{1}{\sqrt{2}}(x - ip)$$



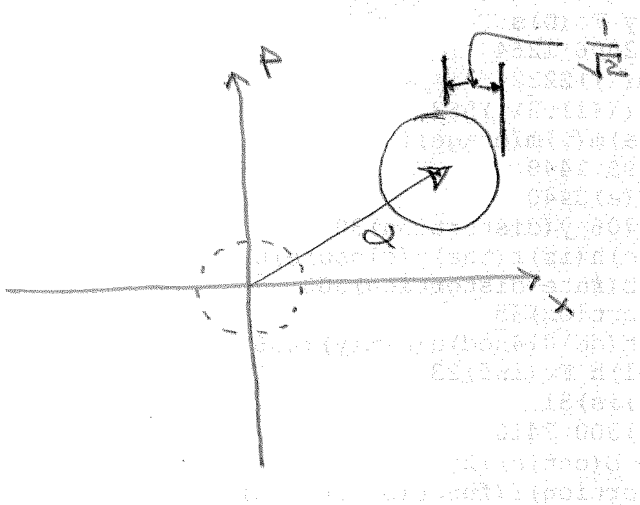


EM field

QM: $[x, p] = i \iff [a, a^{\dagger}] = 1$

$$\Longrightarrow \Delta x \, \Delta p \geq \frac{1}{2}$$

Coherent states



$$\langle a \rangle = \frac{1}{\sqrt{2}}(\langle x \rangle + i \langle p \rangle) = \alpha$$

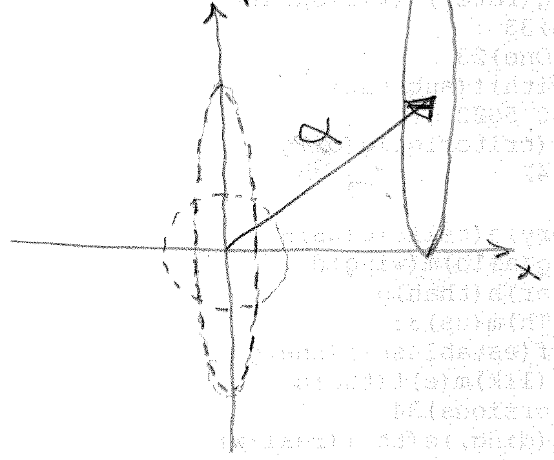
$$(\Delta x)^2 = (\Delta p)^2 = \frac{1}{2}$$

$$|\alpha\rangle = D(\alpha)|0\rangle$$

$$\downarrow$$

$$e^{\alpha a^{\dagger} - \alpha^* a}$$

Squeezed state



$\langle a \rangle = \alpha$

$\Delta x = \frac{1}{\sqrt{2}} e^{-r}$

$\Delta p = \frac{1}{\sqrt{2}} e^{r}$

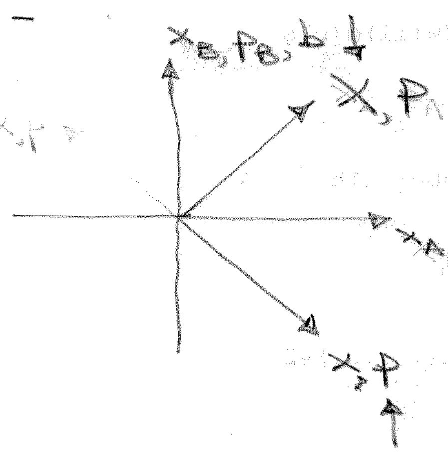
$|\alpha, r\rangle = D(\alpha)\, S(r)\, |0\rangle$

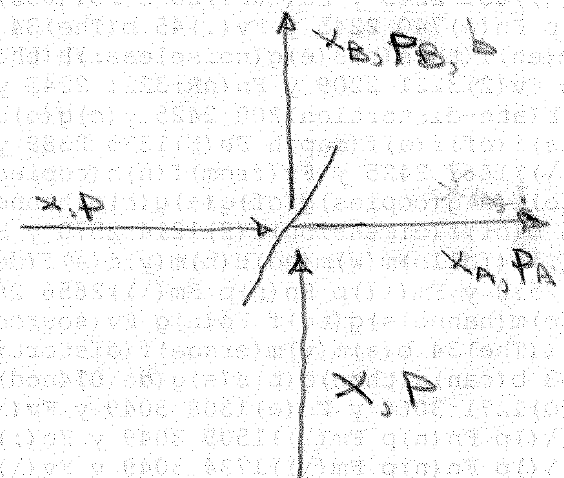
$e^{\frac{1}{2} r (a^2 - a^{\dagger 2})}$

Entanglement:

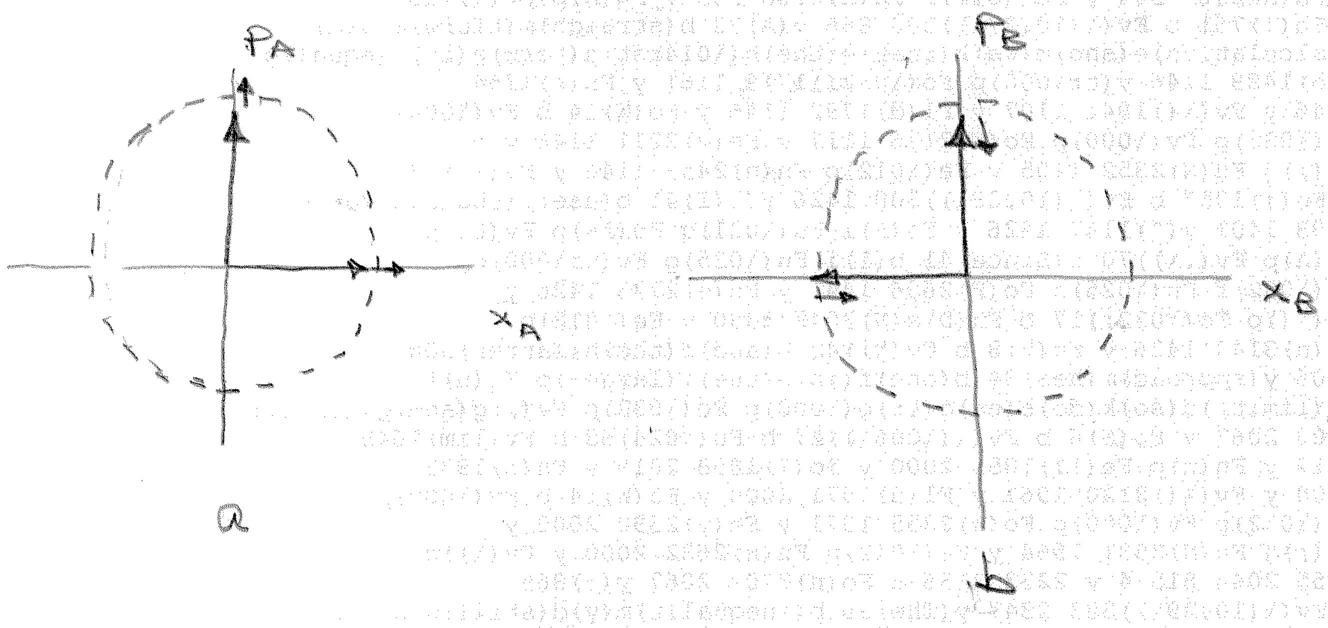
$X = \frac{1}{\sqrt{2}}(x_A + x_B) \qquad \frac{1}{\sqrt{2}}(p_A + p_B) = P$

$x = \frac{1}{\sqrt{2}}(x_A - x_B) \qquad \frac{1}{\sqrt{2}}(p_A - p_B) = p$





Picture



$$S_2(r) = T\, S_1(r) \otimes S_1(-r)\, T^\dagger$$

$$|\psi^{AB}\rangle = S_2(r)\,|0\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty} (\tanh r)^n\, |n\rangle \otimes |n\rangle$$

$$\downarrow$$

$$e^{r(ab - a^\dagger b^\dagger)}$$

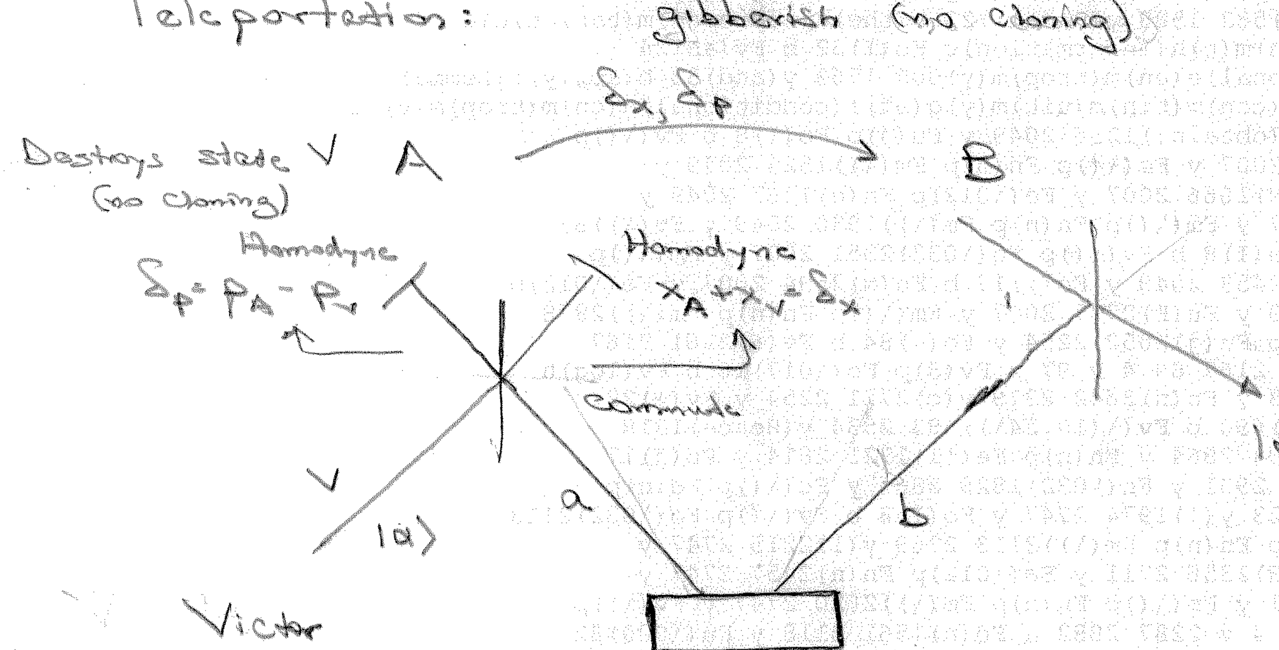
$$S(\rho^A) = S(\rho^B) = c^2 \log c^2 - s^2 \log s^2 \xrightarrow[r \gg 1]{} 2r \log e$$

$$c = \cosh r$$

$$s = \sinh r$$

$$\langle n_A \rangle = \langle n_B \rangle = \frac{1}{2}\cosh 2r \longrightarrow \frac{1}{2} e^{2r}$$

Teleportation:



$$\left.\begin{aligned} x_A &= -x_B \\ p_A &= p_B \end{aligned}\right] \text{ perfect correlation}$$

$$\delta_x = x_A + x_V \qquad x_B' = x_B + \delta_x = -x_A + \delta_x = x_V$$

$$\delta_p = p_A - p_V \qquad p_B' = p_B - \delta_p = p_A - \delta_p = p_V$$

How good can you do classically?

① A measures x_V and p_V; sends result to Bob who makes CS with measured values.

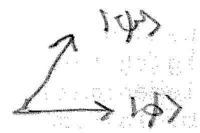
2 units of vacuum noise

(3rd unit in homodyne measurements to compare)

$$\Longrightarrow F = \langle\alpha|\rho|\alpha\rangle \le \frac{1}{2}$$

Q2:

② Transmitting 2 nonorthogonal states



Q3: Work out for

$$V \xrightarrow[|\psi\rangle \text{ or } |\phi\rangle]{} \underset{\text{measurement}}{A} \xrightarrow[\text{classical channel}]{} \underset{\text{prepares new state}}{B} \longrightarrow V$$

$\bar{F} \le f(x)$

$x = |\langle\psi|\phi\rangle|$

③ Wigner function HV model: $F \to 1$ (limit same as in QM for finite squeezing) for CS. Nothing qm in teleporting CS. What about ①? What about ②?

Caves's rule: If it's qm, you can't "understand" it.

or

If you can understand it, it's not qm.

Mixed Gaussian States:

$$\rho^{AB}$$

$$M = \begin{pmatrix} \langle x_A \rangle \\ \langle p_A \rangle \\ \langle x_B \rangle \\ \langle p_B \rangle \end{pmatrix} \qquad C = \begin{pmatrix} \langle (\Delta x_A)^2 \rangle & \overset{\text{Symmetrized}}{\langle \Delta x_A \Delta p_A \rangle} & \langle \Delta x_A \Delta x_B \rangle & \langle \Delta x_A \Delta p_B \rangle \\ & & & \\ & & & \end{pmatrix}$$

Local transformations: $M = 0$

C special form

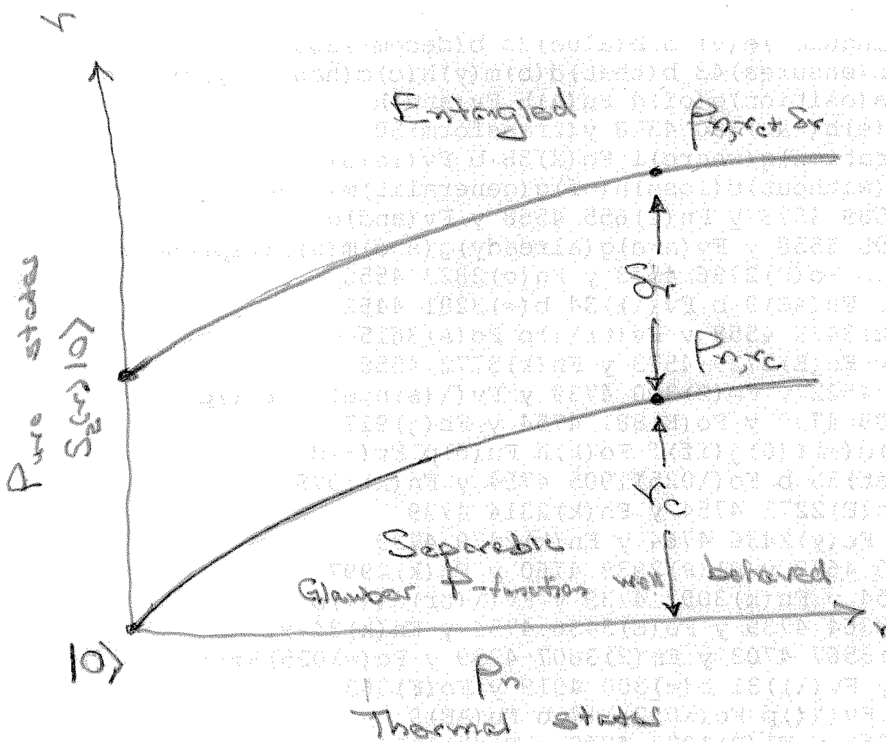
$$\rho^{AB} = S_2(r)\, T\, \underbrace{\rho_{n_A n_B}}_{\text{Thermal in A and B}}\, T^\dagger S_2^\dagger(r)$$

$$C = \begin{pmatrix} a & 0 & c_1 & 0 \\ 0 & a & 0 & c_2 \\ c_1 & 0 & b & 0 \\ 0 & c_2 & 0 & b \end{pmatrix}$$

Simplify slightly: $n_A = n_B = n$

$$\rho^{AB} = S_2(r)\, \rho_n\, S_2^\dagger(r) \equiv \rho_{n,r}$$

$$C_{n,r} = (n+\tfrac{1}{2}) \begin{pmatrix} C & 0 & -S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & S & 0 & C \end{pmatrix} \qquad \begin{aligned} C &= \cosh 2r \\ S &= \sinh 2r \end{aligned}$$



$$\left(n+\tfrac{1}{2}\right)e^{-2r_c} = \frac{1}{2}$$

$$r_c = \frac{1}{2}\ln(2n+1)$$

"Classical squeezing": $D(\mu)\otimes D(-\mu^*)$ Gaussian distribution of μ's

$x_A \uparrow \mu_R \quad x_B \downarrow -\mu_R$

$p_A \uparrow \mu_I \quad p_B \uparrow \mu_I$

$$E(\rho_{n,r_c+\delta r}) \le S(S_2(\delta r)|0\rangle) \equiv E_{min}$$

$$= \cosh^2\delta r \ln\cosh^2\delta r - \sinh^2\delta r \ln\sinh^2\delta r$$

Q3:

But not proven: however, we know we're close because

$$E(\rho_{n,r}) \ge S(\rho^A) - S(\rho^{AB}) \equiv E_{max}$$

$$E_{max} - E_{min} \xrightarrow[\delta r \gg 1]{} 2(1-\ln 2) = 2\ln(e/2) = 2(\log e - 1)$$

nats $\qquad$ bits